Numbering Systems

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

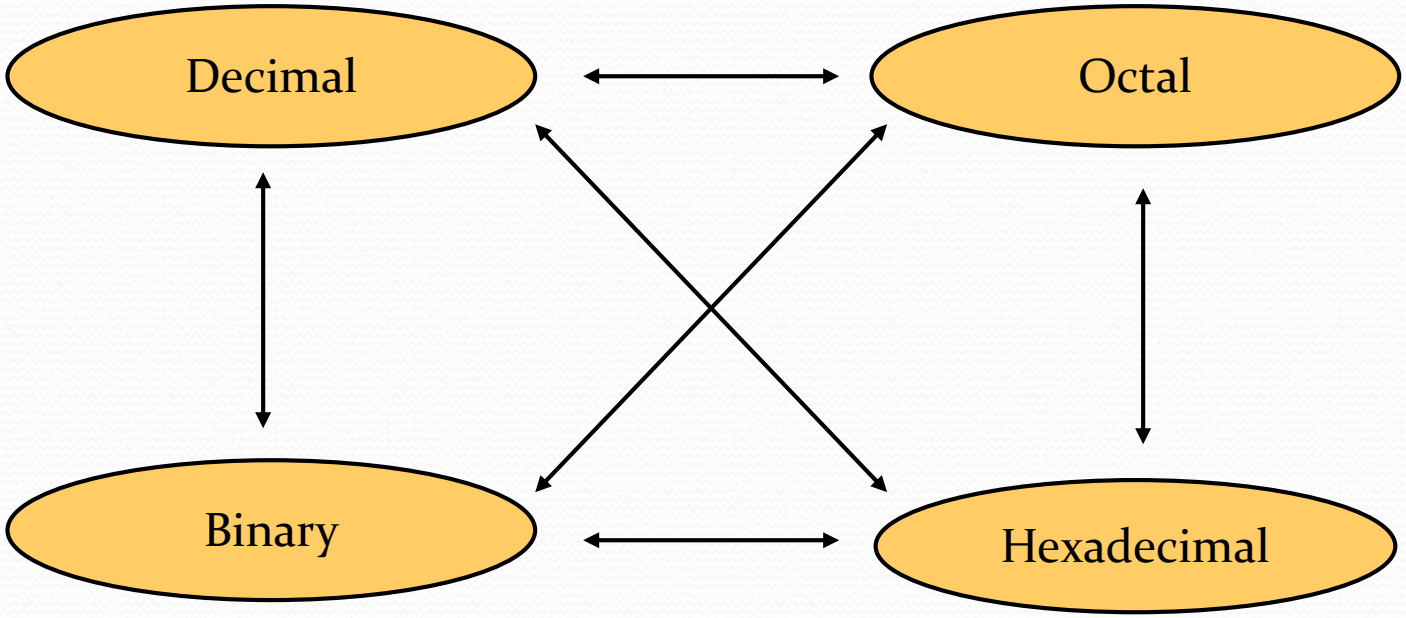
Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

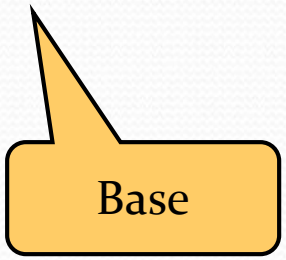
Conversion Among Bases

- The possibilities:

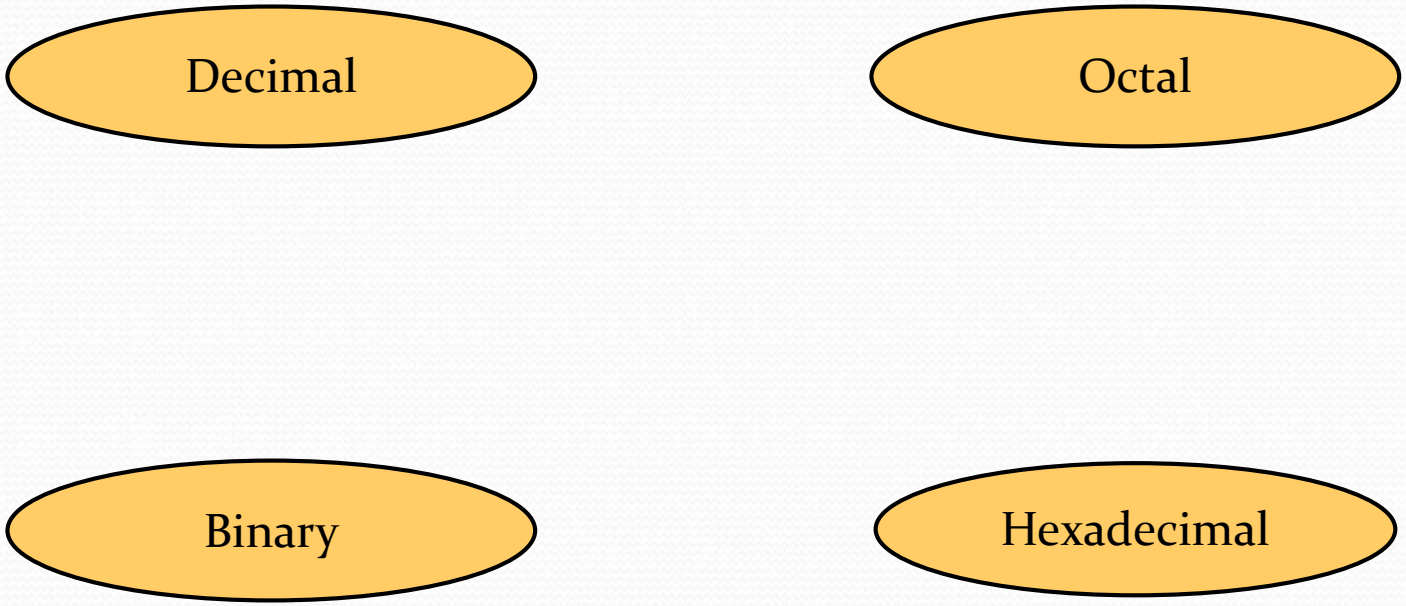


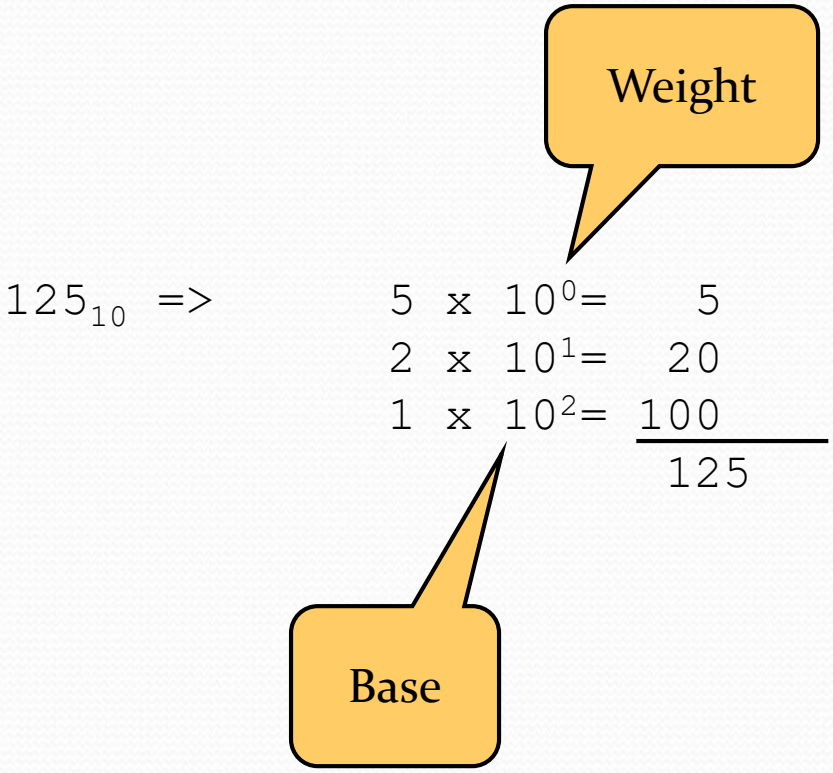
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

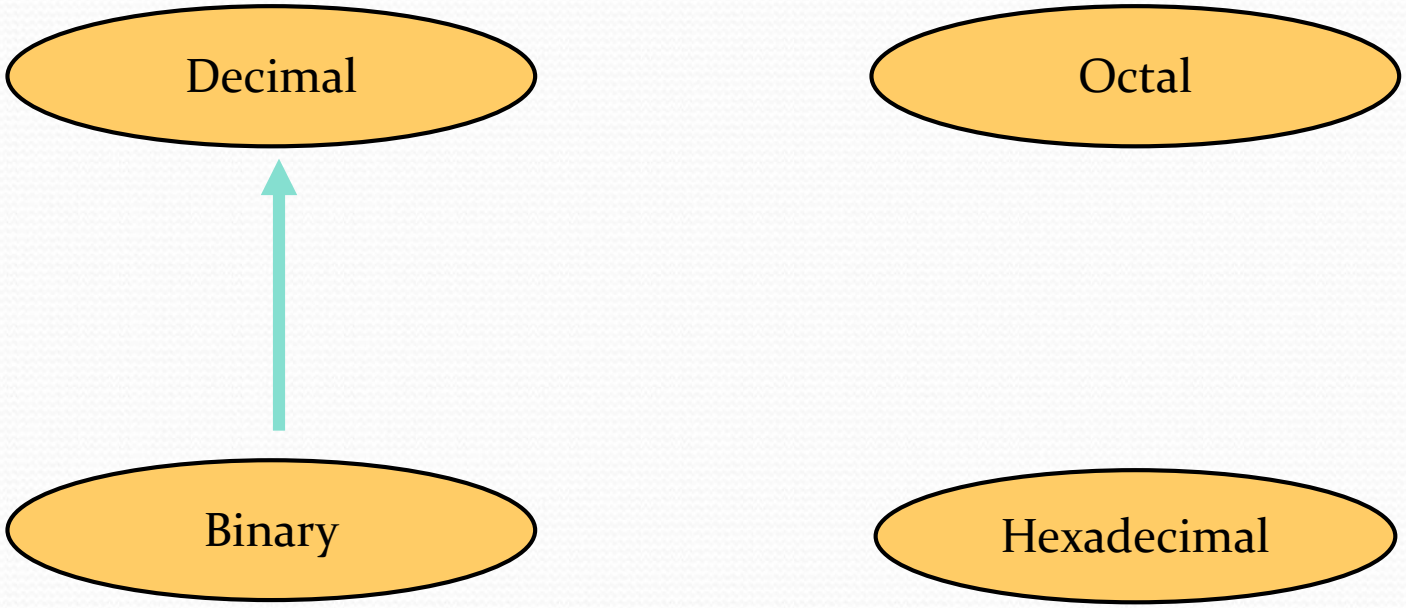


Decimal to Decimal (just for fun)




Weight
125_{10} =>
5 x 10^0= 5
2 x 10^1= 20
1 x 10^2= 100
125
Base

Binary to Decimal



Binary to Decimal

- Technique
  - Multiply each bit by 2^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

Bit “o”

101011_2 =>

$1 \times 2^0 = 1$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

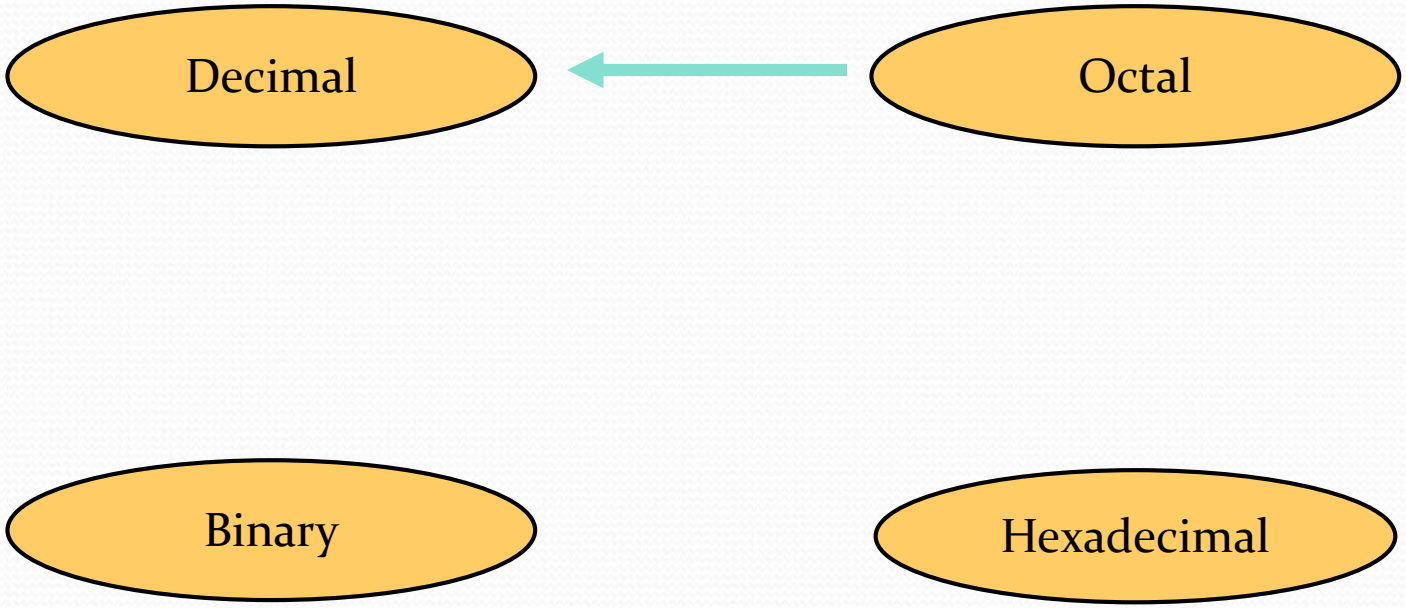
$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$1 \times 2^5 = 32$

43_{10}

Octal to Decimal



Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

724_8 =>

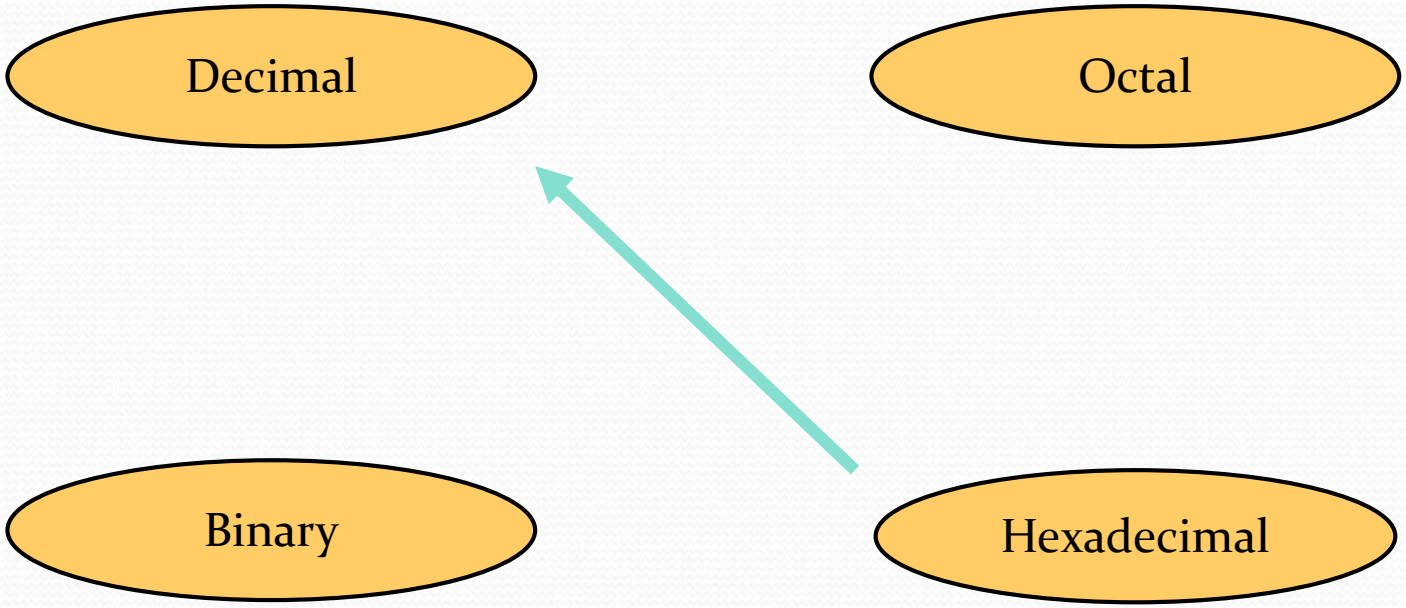
$4 \times 8^0 = 4$

$2 \times 8^1 = 16$

$7 \times 8^2 = 448$

468_{10}

Hexadecimal to Decimal



Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

ABC_{16} =>

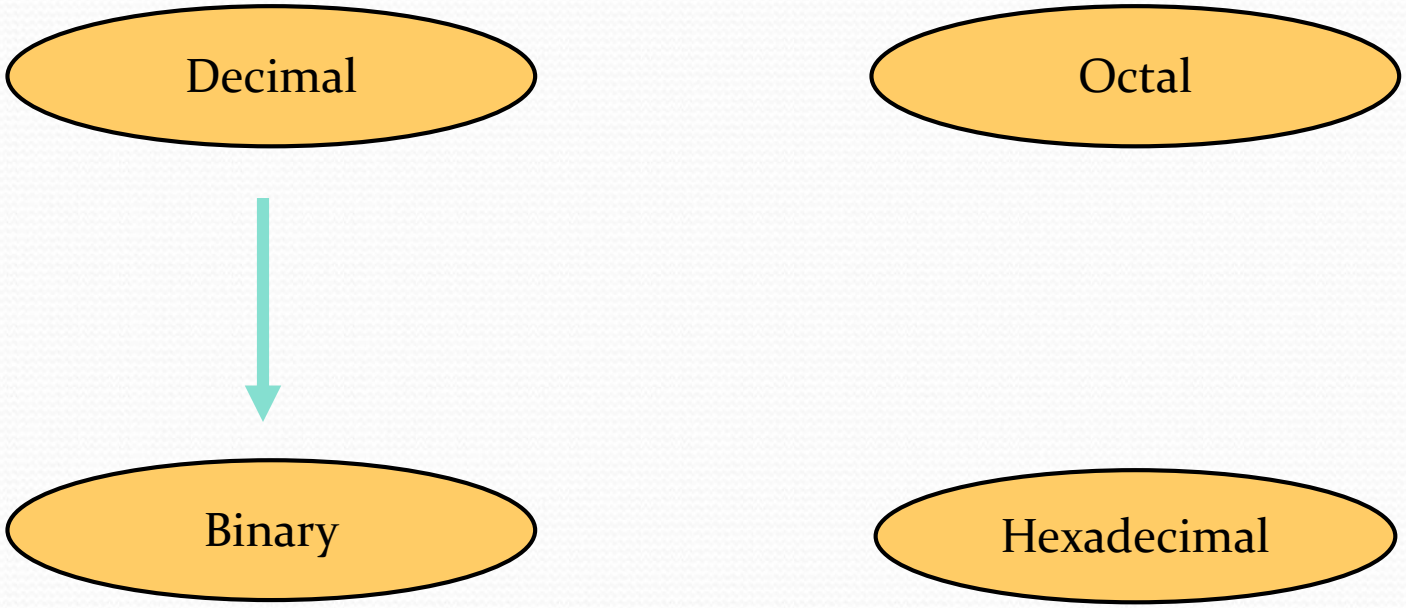
$C \times 16^0 = 12 \times 1 = 12$

$B \times 16^1 = 11 \times 16 = 176$

$A \times 16^2 = 10 \times 256 = 2560$

2748_{10}

Decimal to Binary



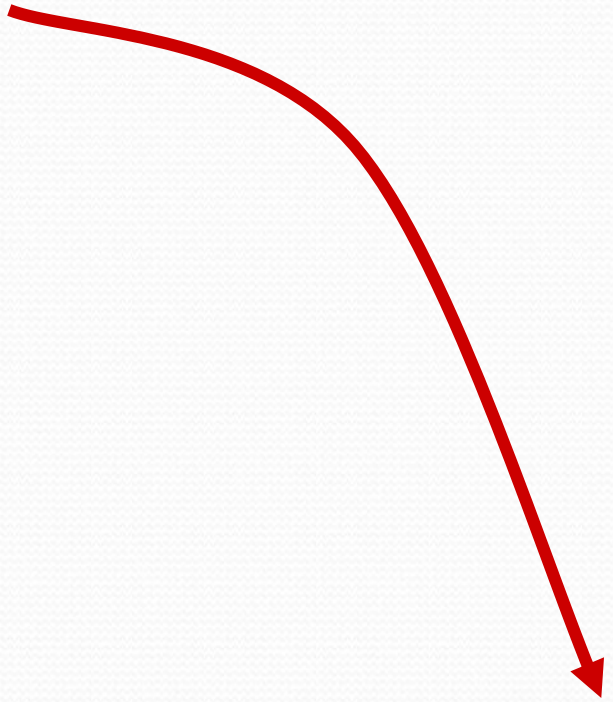
Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

Example

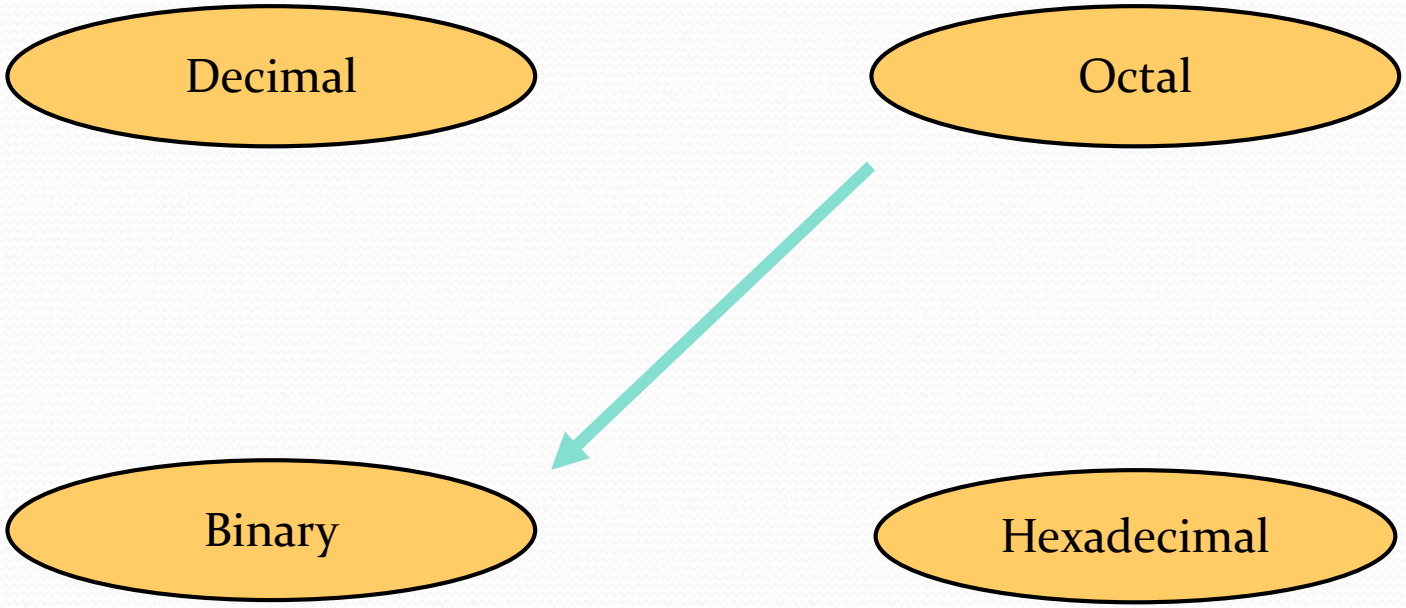
$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1



$125_{10} = 1111101_2$

Octal to Binary

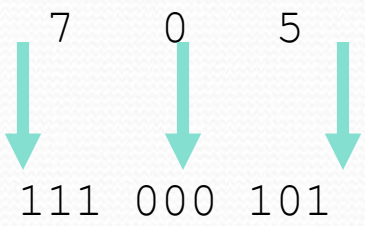


Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

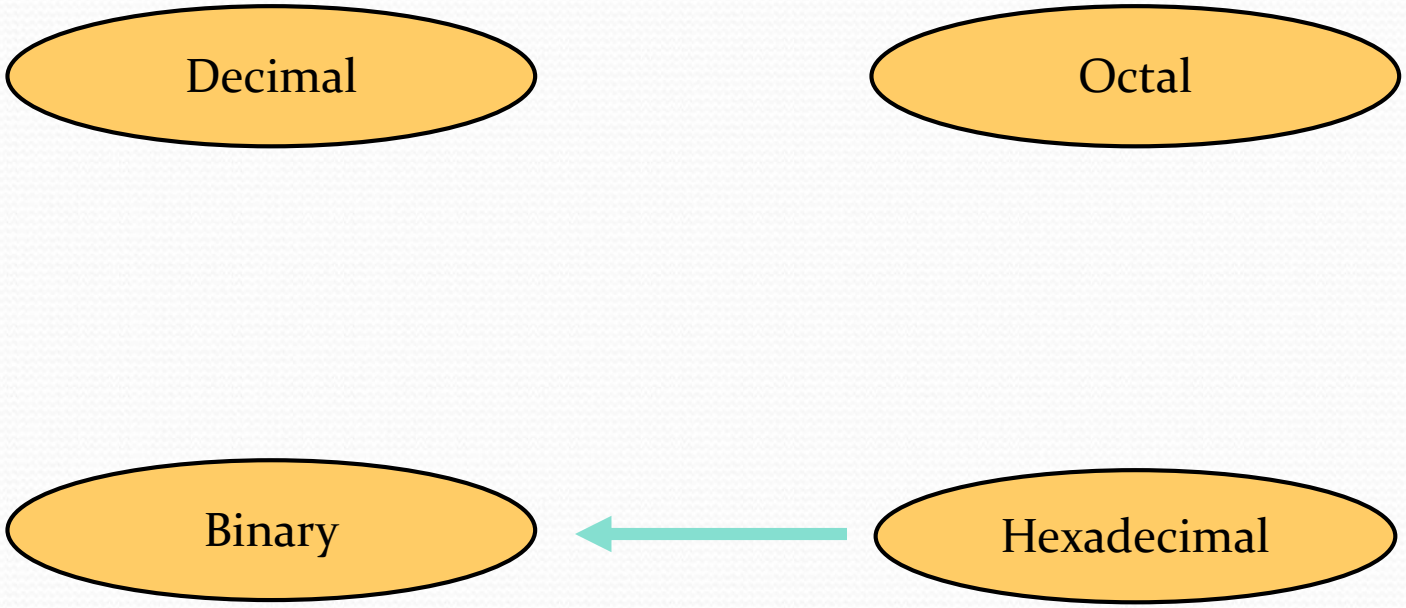
Example

$705_8 = ?_2$



$705_8 = 111000101_2$

Hexadecimal to Binary

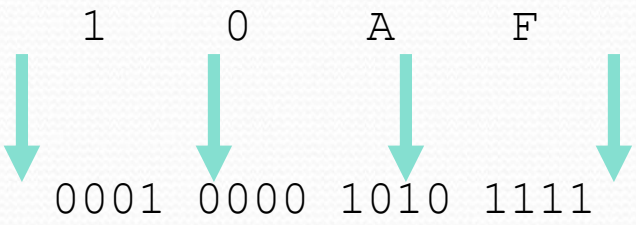


Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

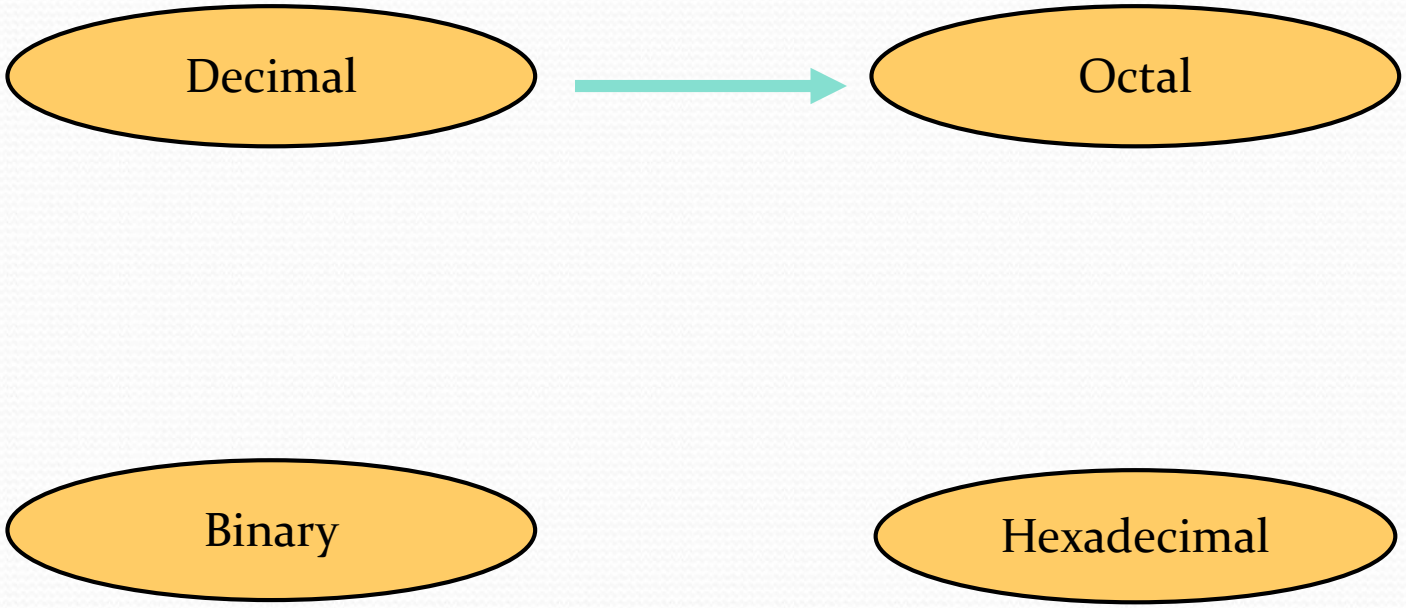
Example

$10AF_{16} = ?_2$



$10AF_{16} = 0001000010101111_2$

Decimal to Octal

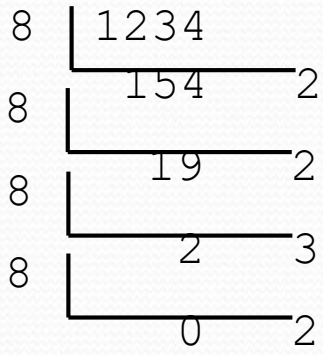


Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

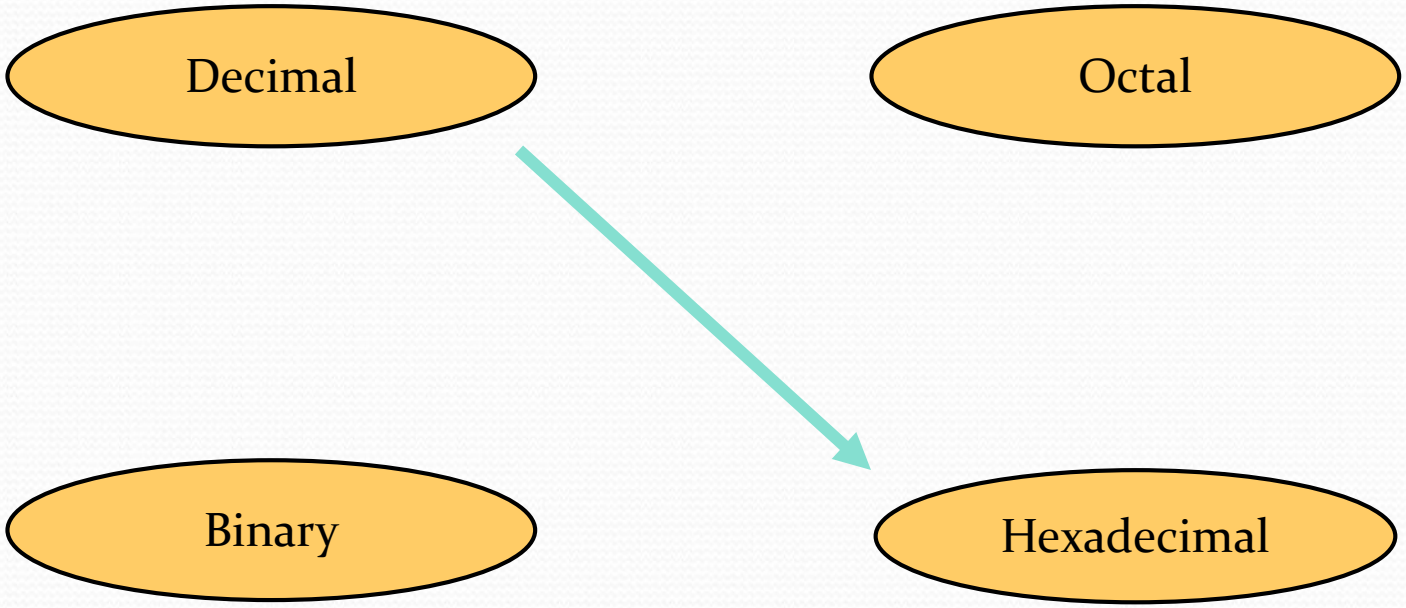
Example

$1234_{10} = ?_{8}$



$1234_{10} = 2322_{8}$

Decimal to Hexadecimal



Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

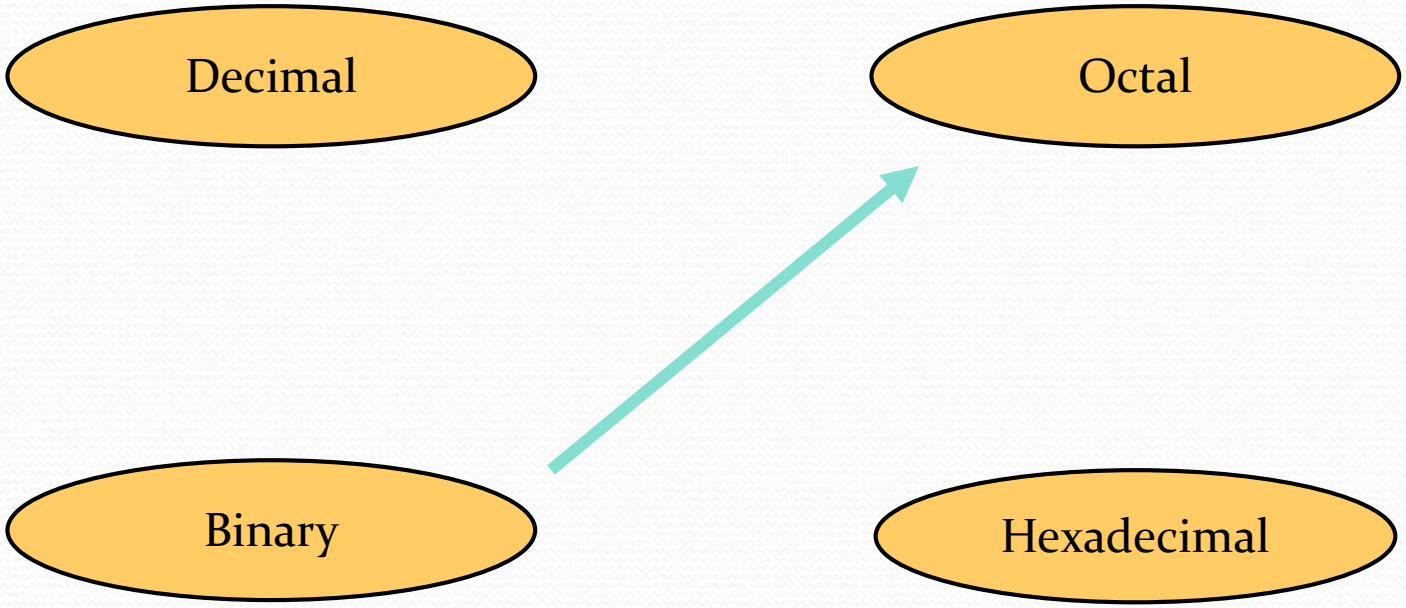
Example

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

Binary to Octal

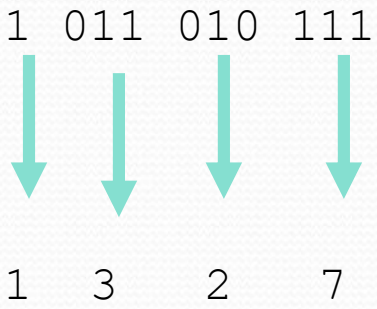


Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

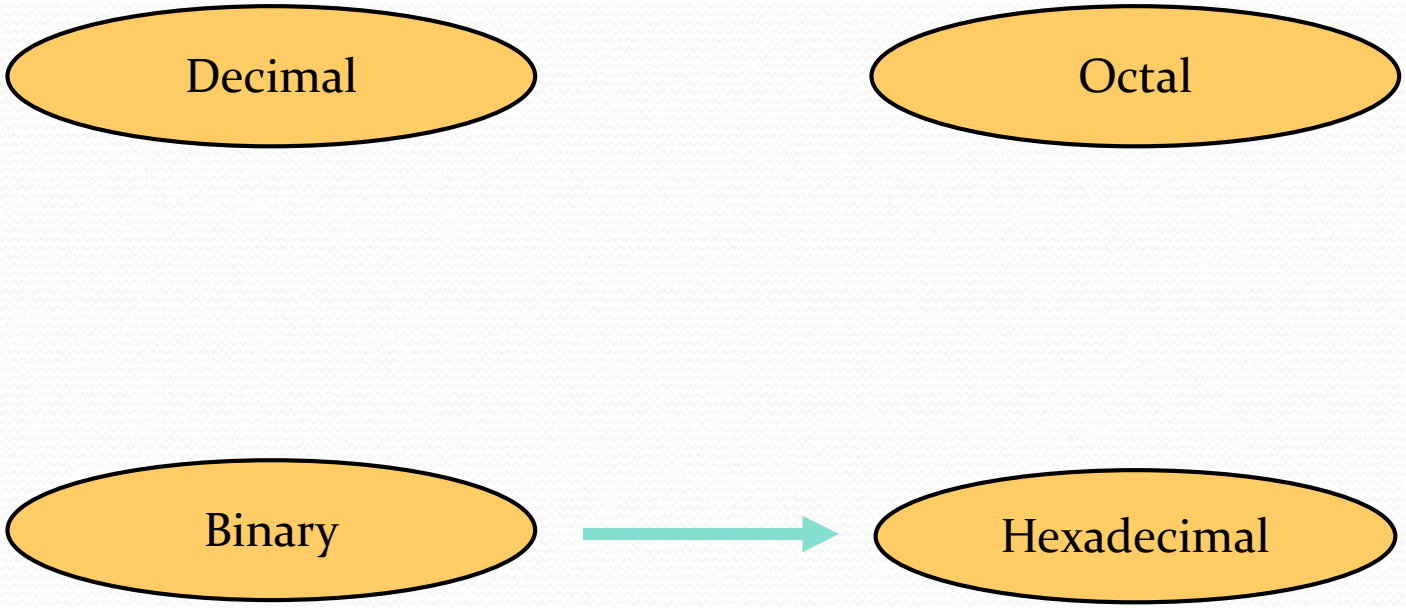
Example

$1011010111_2 = ?_8$



$1011010111_2 = 1327_8$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

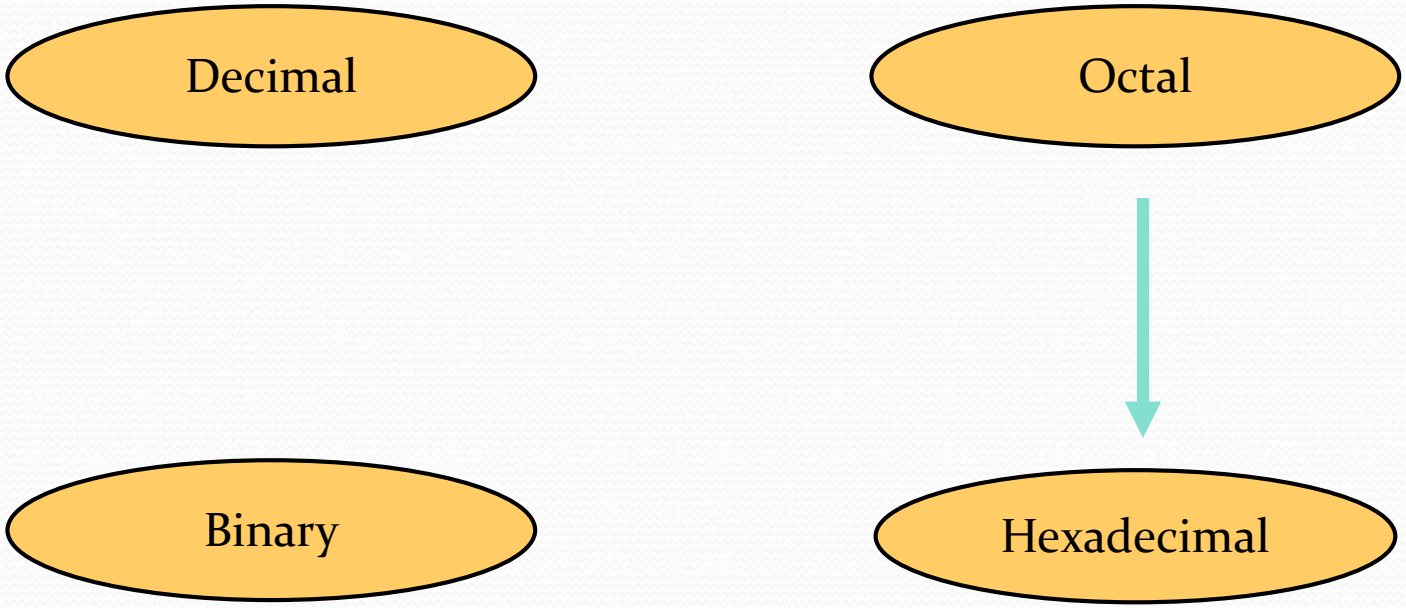
$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal

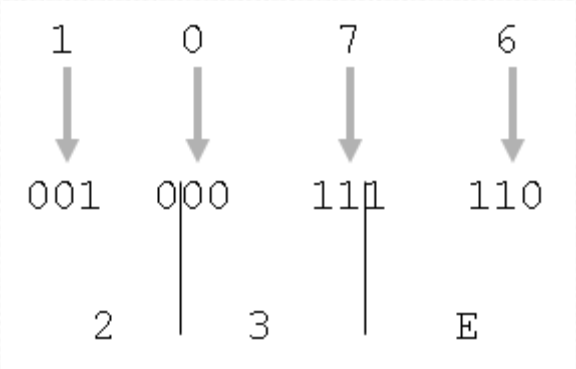


Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

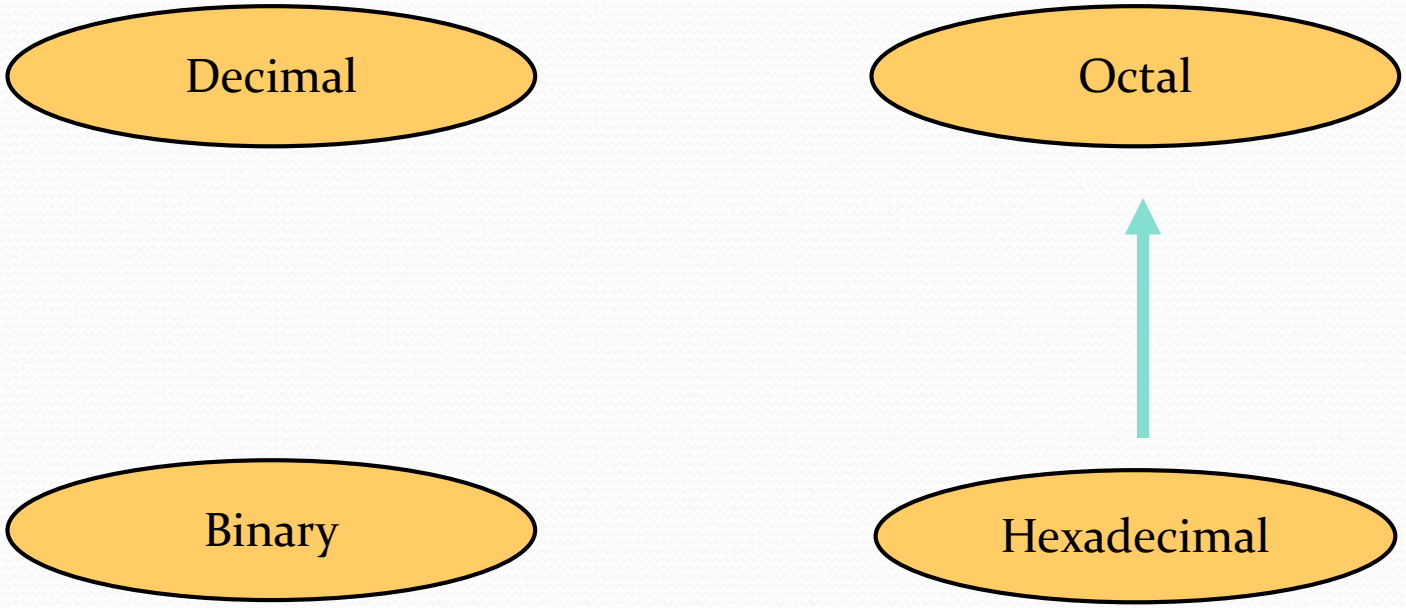
Example

$1076_8 = ?_{16}$



$1076_8 = 23E_{16}$

Hexadecimal to Octal

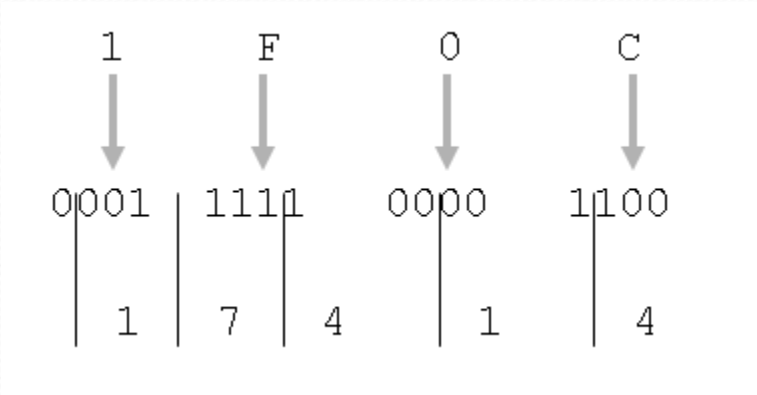


Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

Example

$1F0C_{16} = ?_{8}$



$1F0C_{16} = 17414_{8}$

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Common Powers (1 of 2)

- Base 10

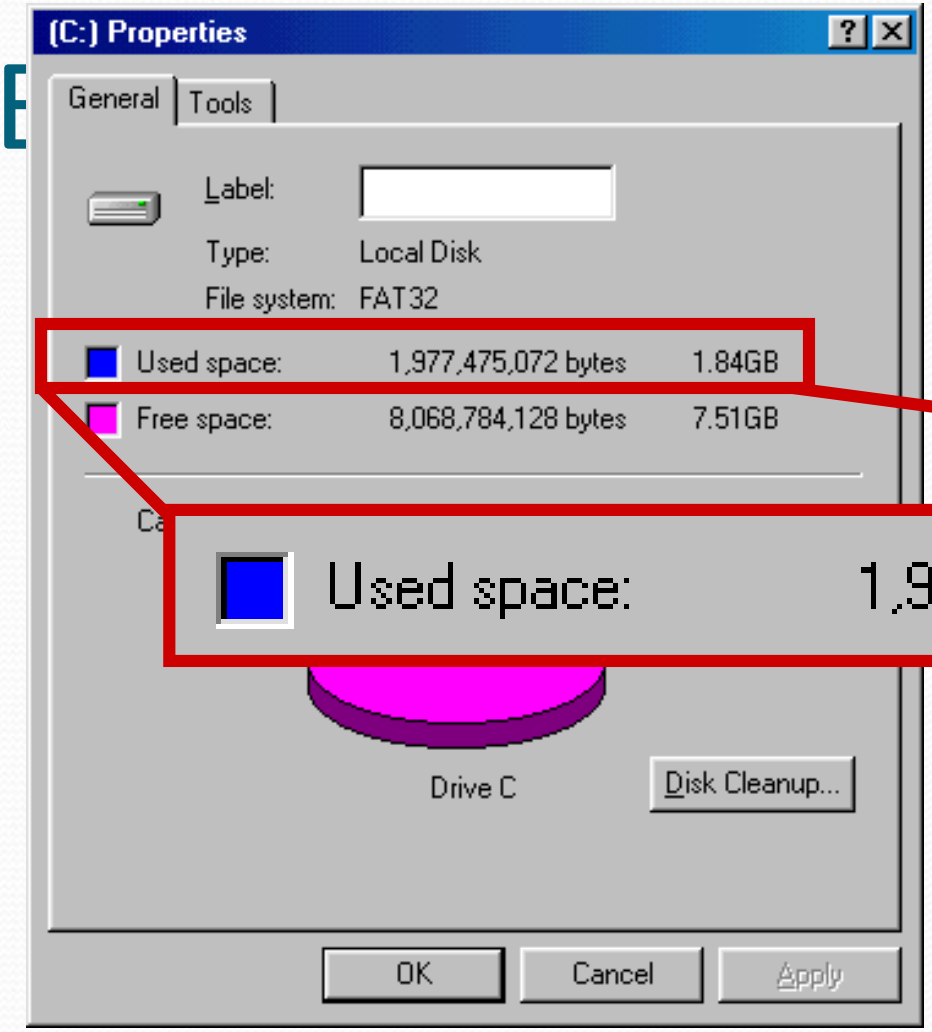
Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	μ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

Common Powers (2 of 2)

- Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of “k”, “M”, and “G”?
- In computing, particularly w.r.t. memory, the base-2 interpretation generally applies



In the lab…

1. Double click on My Computer
2. Right click on C:
3. Click on Properties

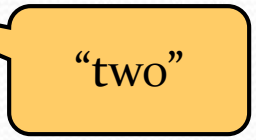
Used space: 1,977,475,072 bytes 1.84GB

/ 2^{30} =

Binary Addition (1 of 2)

- Two 1-bit values

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10



Binary Addition (2 of 2)

- Two *n*-bit values
  - Add individual bits
  - Propagate carries
  - E.g.,

```
 1    1
  10101        21
+ 11001      + 25
-------      ----
 101110        46
```